A characterization method for the STSP based on Frequency K_is

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The symmetric traveling salesman problem (STSP) is generally represented as a weighted complete graph K_n. Because the edges in optimal Hamiltonian cycle (OHC) are not precisely characterized by the weights on edges, the computational complexity of TSP is hard to reduce. To better characterize STSP, the frequency K_is are proposed to compute the frequency of an edge. One frequency K_i is computed with the $\binom{i}{2}$ optimal i-vertex paths with given endpoints within a given weighted K_i in K_n. Different from the weights of edges in weighted K_i, the frequencies of OHC edges and those of ordinary edges in frequency K_i have much difference. The lower bound of the expected frequency for OHC edges is $\frac{1}{2}\binom{i}{2}$, whereas the upper bound of the expected frequency for ordinary edges is 3(n-2).

As the frequency of an edge in K_n is computed based on the frequency K_is containing it, we proved that the lower bound of the expected frequency for OHC edges is $\frac{1}{2}\binom{i}{2}$. It means that an OHC edge in K_n is the OHC edge in every K_i containing it on average. Moreover, the probability that an OHC edge is contained in the optimal i-vertex paths with given endpoints increases or is permitted to decrease in proportion to a factor smaller than $1-\frac{2}{i(i-1)}$ from i to i+1∈[5,n]. For an ordinary edge, the probability that it is contained in the optimal i-vertex paths with given endpoints decreases according to i in the average case. It also found that the probability that an ordinary edge is contained in the optimal i-vertex paths with given endpoints decreases from i to i+1 if i meeting the inequality $\frac{(n-2)(n-3)-(i-2)(i-3)}{(n-2)(n-3)-(i-1)(i-2)} \geq \sqrt{1+\frac{2}{i(i+1)}}$. The numerical simulation demonstrated that the first $i=O(n^{\frac{4}{7}})$ which is much smaller than n. Based on the change of the probabilities, all ordinary edges can be found with respect to the optimal i-vertex paths with given endpoints containing $O(n^{\frac{4}{7}})$ vertices. It means that STSP can be resolved in $O\left(2^{O\left(n^{\frac{4}{7}}\right)}\right)$ time based on dynamic programming.

Note: If an edge (i,j) is taken as a variable x_{ij}, the weight w_{ij} of (i,j) is a function $w_{ij}=g(x_{ij})$. Since the frequency of (i,j) is computed based on the optimal i-vertex paths or frequency K_is, the edge frequency $f_{ij}=F(w_{ij})= F[g(x_{ij})]$ is a functional with respect to the edge weight function $w_{ij}=g(x_{ij})$. Thus, the OHC edges and ordinary edges are more precisely characterized using the frequencies of edges.